Holly Lodge Primary School

Calculation Policy for Mathematics

How we teach calculations

Building Together for the future!

About our Calculation Policy

The following calculation policy has been devised to meet requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is also designed to give pupils a consistent and smooth progression of learning in calculations across the school. Please note that early learning in number and calculation in Foundation Stage follows the *'Development Matters'* EYFS document, and this calculation policy is designed to build on progressively from the content and methods established in the Early Years Foundation Stage.

Age/stage expectations

The calculation policy is organised according to age expectations as set out in the National Curriculum 2014. Most children will work at the stage relating to their year group e.g. Stage 5 in Year 5. However, some children may work at a lower level until they are ready to move on. A few children may move onto a higher level than the rest of their year group but only if they have already achieved 'mastery' of that level (i.e. if they have a very deep understanding of mathematical concepts).

Providing a context for calculation:

It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods when faced with problems.

Choosing a calculation method:

Children need to be taught and encouraged to use the following processes in deciding what approach they will take to a calculation, to ensure they select the most appropriate method for the numbers involved:

Can I do it in my head using a mental strategy?

Could I use some jottings to help me?

Should I use a written method to work it out?

The Number System and Place Value

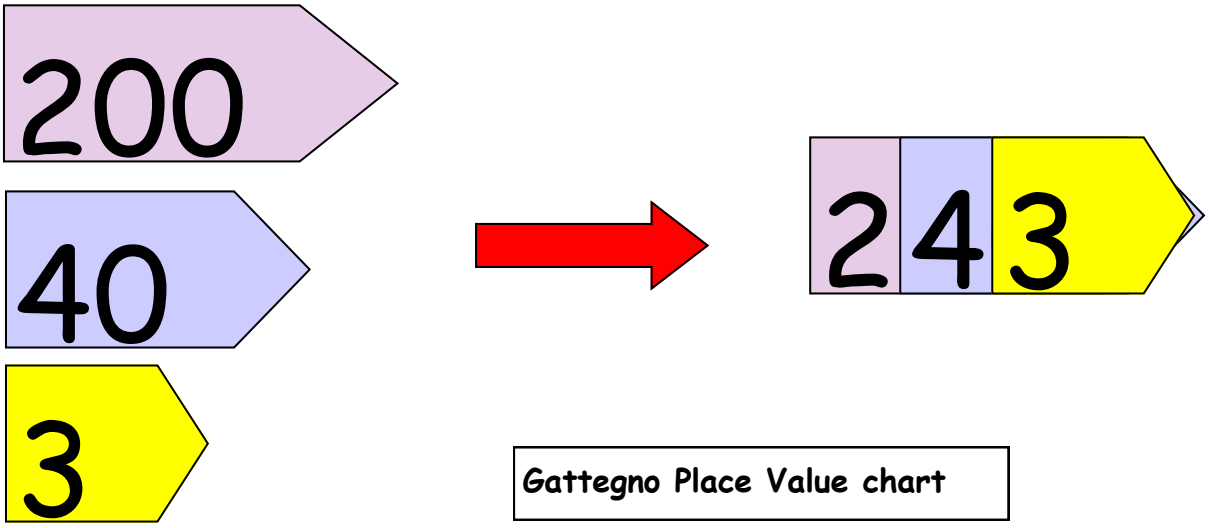
A full understanding of place value within the number system is the foundation on which mathematical development is built. A range of resources are used to develop children's understanding of this fundamental concept.

Place Value Charts used with place value counters (or Base 10 or money)

100s	10s	1s
100 100	10 10 10 10	1 1 1

2 hundreds	+ 4 tens	+ 3 units	= 243
200	+ 40	+ 3	= 243

Place Value Arrows



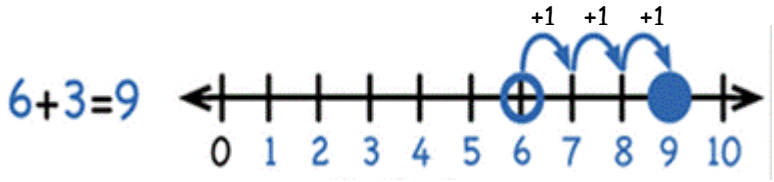
Gattegno Place Value chart

0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900

Addition

Stage 1 Add with numbers up to 20

Use numbered number lines to add, by counting on in ones. Encourage children to start with the **larger** number and count on.



Children should:

- Have access to a wide range of counting equipment, everyday objects, number tracks and number lines, and be shown numbers in different contexts.
- Read and write the addition (+) and equals (=) signs within number sentences.
- Interpret addition number sentences and solve missing box problems, using concrete objects and number line addition to solve them: 8 + 3 = □ 15 + 4 = □ 5 + 3 + 1 = □ □ + □ = 6

This builds on from prior learning of adding by combining two sets of objects into one group (5 cubes and 3 cubes) in Early Years.

Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.

8 + 5

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line

Key skills for addition at Stage 1:

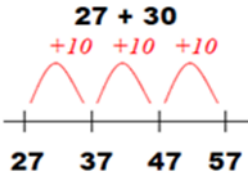
- Read and write numbers to 100 in numerals, incl. 1—20 in words
- Recall bonds to 10 and 20, and addition facts within 20
- Count to and across 100
- Count in multiples of 1 2, 5 and 10
- Solve simple 1-step problems involving addition, using objects, number lines and pictorial representations.
- Read, write and interpret mathematical statements including addition (+) and equals (=) signs.

Addition

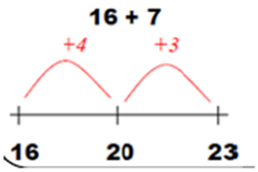
Stage 2 Add with 2-digit numbers

Developing mental fluency with addition and place value involving 2-digit numbers, then establish more formal methods.

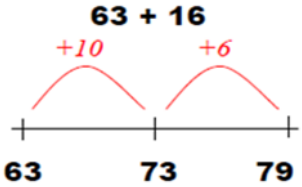
Add 2 digit numbers and tens:



Add 2 digit numbers and units:



Add pairs of 2 digit numbers, moving to the partitioned column method when secure adding tens and units:



23 + 34:

	2	0 +	3	
+	3	0 +	4	
	5	0 +	7	
			=	57

STEP 1: Only provide examples that do **NOT** cross the tens boundary until they are secure with the method itself.

STEP 2: Once children can add a multiple of ten to a 2-digit number mentally (e.g. 80+11), they are ready for adding pairs of 2-digit numbers that DO cross the tens boundary (e.g. 58 + 43).

5	0 +	8		
4	0 +	3		
9	0 +	1	1	
			=	101

STEP 3: Children who are accurate with this stage should move onto expanded addition methods with 2 and 3-digit numbers (see Y3).

To support understanding, children may physically make and carry out the calculation with place value counters or other resources, then compare their practical version to the written form, to help them to build an understanding of it.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, **sum, tens, units, partition, addition, column, tens boundary**

Key skills for addition at Stage 2:

- Add a 2-digit number and ones (e.g. 27 + 6) or tens (e.g. 23 + 40)
- Add pairs of 2-digit numbers (e.g. 35 + 47)
- Add three single-digit numbers (e.g. 5 + 9 + 7)
- Show that adding can be done in any order (the commutative law).
- Recall bonds to 20 and bonds of tens to 100 (30 + 70 etc.)
- Count in steps of 2, 3 and 5 and count in tens from any number.
- Understand the place value of 2-digit numbers (tens and ones)
- Compare and order numbers to 100 using < > and = signs.
- Read and write numbers to at least 100 in numerals and words.
- Solve problems with addition, using concrete objects, pictorial representations, involving numbers, quantities and measures, and applying mental and written methods.

Addition

Stage 3 Add numbers with up to 3-digits

Introduce the **expanded column addition** method:

	2	3	6
+		7	3
			9
	1	0	0
	2	0	0
	3	0	9

Add the units first, in preparation for the compact method.

In order to carry out this method of addition:

- Children need to recognise the value of the hundreds, tens and units without recording the partitioning.
- Pupils need to be able to add in columns.

Move to the compact **column addition** method, with 'carrying':

Children who are very secure and confident with 3-digit expanded column addition should be moved onto the **compact column addition** method, being introduced to 'carrying' for the first time. Compare the expanded method to the compact column method to develop an understanding of the process and the reduced number of steps involved.

Add units first

```
   236
+   73
   309
    1
```

'Carry' numbers **underneath** the bot-

Remind pupils that the actual value is 'three tens plus sevens tens', not three add seven.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, **hundreds boundary, increase, vertical, 'carry', expanded, compact**

Key skills for addition at Stage 3:

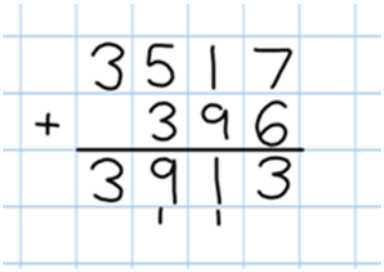
- Read and write numbers to 1000 in numerals and words.
- Add 2-digit numbers mentally, including those exceeding 100.
- **Add a three-digit number and ones mentally (175 + 8)**
- **Add a three-digit number and tens mentally (249 + 50)**
- **Add a three-digit number and hundreds mentally (381 + 400)**
- Estimate answers to calculations, using inverse to check answers.
- Solve problems, including missing number problems, using number facts, place value, and more complex addition.
- Recognise place value of each digit in 3-digit numbers (hundreds, tens, ones..
- Continue to practise a wide range of mental addition strategies, ie. number bonds, adding the nearest multiple of 10, 100, 100 and adjusting, using near doubles, partitioning and recombining.

Addition

Stage 4 Add numbers with up to 4 digits

Move from expanded addition to the compact column method, **adding units first**, and 'carrying' numbers **underneath** the calculation.

e.g. 3517 + 396 = 3913



Introduce the **compact column addition** method by asking children to add the two given numbers together using the method that they are familiar with (expanded column addition—see Y3). Teacher models the compact method with carrying, asking children to discuss similarities and differences and establish how it is carried out.

Add units first

'Carry' numbers **underneath** the bottom line.

Remind pupils that the actual value is 'five hundreds add three hundreds' not five add three.

Apply this method to contexts such as measures and money, including 2-step problems.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact, **thousands, hundreds, digits, inverse**

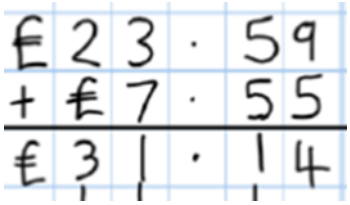
Key skills for addition at Stage 4:

- Select most appropriate method: mental, jottings or written and explain why.
- Recognise the place value of each digit in a four-digit number.
- Round any number to the nearest 10, 100 or 1000.
- Estimate and use inverse operations to check answers to a calculation.
- Solve 2-step problems in context, deciding which operations and methods to use and why.
- Find 1000 more or less than a given number.
- Continue to practise a wide range of mental addition strategies, ie. number bonds, add the nearest multiple of 10, 100, 1000 and adjust, use near doubles, partitioning and recombining.
- Add numbers with up to 4 digits using the formal written method of column addition.

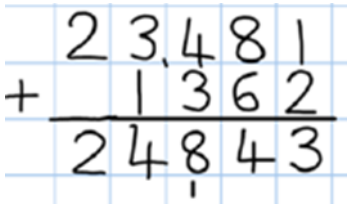
Addition

Stage 5 Add numbers with more than 4 digits

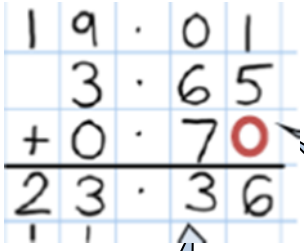
including money, measures and decimals with different numbers of decimal places.



The decimal point should be aligned in the same way as the other place value columns, and must be in the same column in the answer.



Numbers should exceed 4 digits.



Pupils should be able to add more than two values, carefully aligning place value columns.

'Say '6 tenths add 7 tenths 'to reinforce place value.

Empty decimal places can be filled with zero to show the place value in each column.

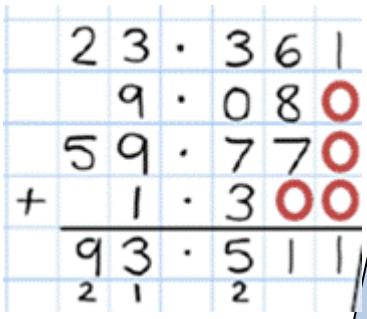
Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, „carry", expanded, compact, vertical, thousands, hundreds, digits, inverse & **decimal places, decimal point, tenths, hundredths, thousandths**

Key skills for addition at Stage 5:

- Add numbers mentally with increasingly large numbers, using and practising a range of mental strategies e.g. add the nearest multiple of 10, 100, 100 and adjust; use near doubles, inverse, partitioning and re-combining; using number bonds.
- Use rounding to check answers and accuracy.
- Solve multi-step problems in contexts, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000.
- Add numbers with more than 4 digits using formal written method of columnar addition.

Addition

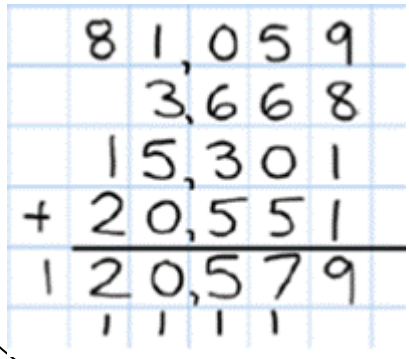
Stage 6 Add several numbers of increasing complexity



Adding several numbers with different numbers of decimal places (including money and measures):

- Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.
- Zeros could be added into any empty decimal places, to show there is no value to add.

Empty decimal places can be filled with zero to show the place value in each column.



Adding several numbers with more than 4 digits.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, 'carry', expanded, compact, vertical, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths

Key skills for addition at Stage 6:

- Perform mental calculations, including with mixed operations and large numbers, using and practising a range of mental strategies.
- Solve multi-step problems in context, deciding which operations and methods to use and why.
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- Round any whole number to a required degree of accuracy.
- Use negative numbers in context and calculate intervals across zero.

Subtraction

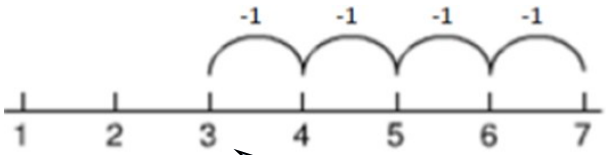
Stage 1 Subtract from numbers up to 20

Children consolidate understanding of subtraction practically, showing subtraction on bead strings, using cubes etc. and in familiar contexts, and are introduced to more formal recording using number lines as below:

Use Cuisenaire rods for subtraction too

Subtract by taking away

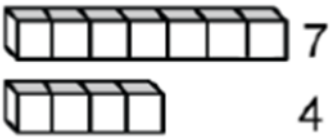
Count back in ones on a numbered number line to take away, with numbers up to 20:



Model counting back using hundred squares, number tracks/lines and practical apparatus such as cubes.

Find the 'distance between'

This will be introduced practically with the language 'find the distance between' and 'how many more' in a range of familiar contexts.



Seven is three more than four.

I am three years older than my sister.

Mental Subtraction

Children should start recalling subtraction facts up to and within 10 and 20, and should be able to subtract zero.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_?

Key skills for subtraction at Stage 1:

- Given a number, say **one more or one less**.
- Count to and over 100, **forward and back**, from any number.
- Represent and use **subtraction facts to 20 and within 20**.
- Subtract with **one-digit and two-digit** numbers to 20, including zero.
- Solve one-step problems that involve addition and subtraction, using concrete objects (ie bead string, objects, cubes) and pictures, and missing number problems.
- Read and write numbers from 0 to 20 in numerals and words.
- Read, write and interpret mathematical statements including subtraction (-) and equals (=) signs.

Subtraction

Stage 2 Subtract with 2-digit numbers

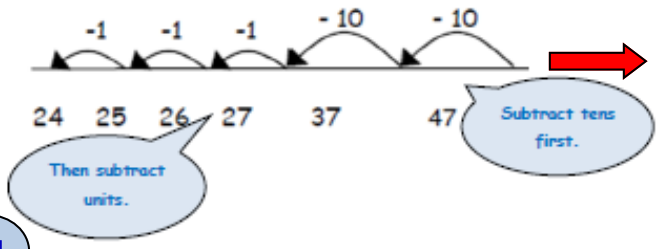
Subtract on a number line by counting back, aiming to develop mental subtraction skills.

This strategy will be used for:

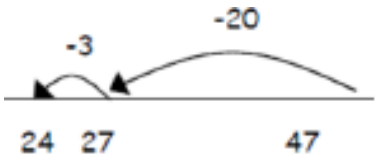
- **2-digit numbers subtract units** (by taking away / counting back) e.g. 36 - 7
- **2-digit numbers subtract tens** (by taking away / counting back) e.g. 48 - 30
- **Subtracting pairs of 2-digit numbers** (see below:)

Subtracting pairs of 2-digit numbers on a number line

47 - 23 = 24 Partition the second number and subtract it in tens and units, as below:

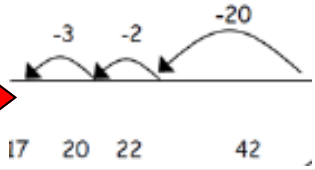


Move towards more efficient jumps back, as below:



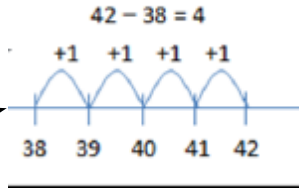
These methods will combine with use of a hundred square to support understanding

Teaching children to **bridge through ten** can help them to become more efficient, for example 42—25:



Mental strategy - subtract numbers close together by **counting on:**

Start with the smaller number and count on to the largest



Many mental strategies are taught. Children are taught to recognise that when numbers are close together, it is more efficient to **count on** the difference. They need to be clear about the relationship between addition and subtraction.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? **difference, count on, strategy, partition, tens, units**

Key skills for subtraction at Stage 2:

- Recognise the place value of each digit in a two-digit number.
- Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100.
- Subtract using concrete objects, pictorial representations, 100 squares and mentally, including: a two- digit number and ones, a two-digit number and tens, and two two-digit numbers.
- Show that subtraction of one number from another cannot be done in any order.
- Recognise and use inverse relationship between addition and subtraction, using this to check calculations and missing number problems.
- Solve simple addition and subtraction problems including measures, using concrete objects, pictorial representation, and also applying their increasing knowledge of mental and written methods.
- Read and write numbers to at least 100 in numerals and in words.

Subtraction

Stage 3 Subtracting with 2 and 3-digit numbers

Introduce **partitioned column subtraction** method.

STEP 1: introduce this method with examples where **no exchanging** is required.

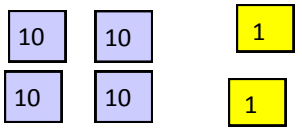
89 - 35 = 54

80 + 9
- 30 + 5
50 + 4

When learning to exchange, explore partitioning in different ways so that children understand that when you exchange the VALUE is the same e.g. 42 = 40 + 2 = 30 + 12 = 20 + 22 etc. Emphasise that the value hasn't changed, we have just partitioned it in a different way.

STEP 2: introduce 'exchanging' through practical subtraction. Make the larger number using place value counters then subtract 18 from it.

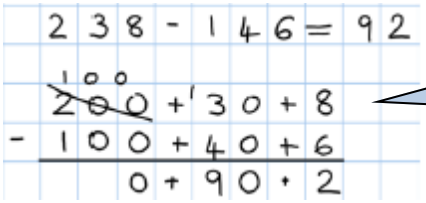
42 - 18



30 ~~40~~ + 12
-10 + 8
20 + 4 = 24

Before subtracting 8 from the 42 blocks, they will need to exchange a ten counter for ten units.

STEP 3: Once pupils are secure with the concept of 'exchanging', they can use the partitioned column method to subtract 2 and 3 digit numbers.



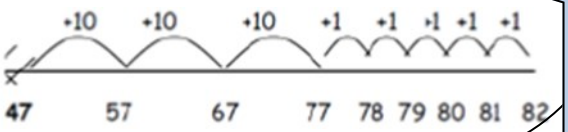
Subtracting money, partition into e.g. £1 + 30p + 8p

Some children will move onto the compact method (see Stage 4)

Counting on as a mental strategy for subtraction

Continue to reinforce counting **on** as a strategy for **close-together** numbers (e.g. 121—118), and also for numbers that are 'nearly' multiples of 10, 100, 100 or £s, which make it easier to count on (e.g. 102—89, 131—79, calculating change from £1 etc.).

Start with the smaller number and count on in tens first, then count on in units to find the rest of the difference.



Counting on in tens is the way we use a hundred square

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units, **exchange, decrease, hundreds, value, digit**

Key skills for subtraction at Stage 3:

- Subtract mentally a: **3-digit number and ones, 3-digit number and tens, 3-digit number and hundreds.**
- Estimate answers and use inverse operations to check.
- Solve problems, including missing number problems.
- Find 10 or 100 more or less than a given number.
- Recognise the place value of each digit in a 3-digit number.
- Counting up differences as a mental strategy when numbers are close together or near multiples of 10 (see examples above)
- Read and write numbers up to 1000 in numerals and words.
- Practise mental subtraction strategies, such as subtracting near multiples of 10 and adjusting (e.g. subtracting 19 or21), and select most appropriate methods to subtract, explaining why.
- Subtract numbers with up to three digits using (partitioned or compact) column subtraction.

Subtraction

Stage 4 Subtract with up to 4-digit numbers

Partitioned column subtraction with exchanging (decomposition):

2754 − 1562 = 1192

600

2000 + ~~700~~ + 150 + 4

− 1000 + 500 + 60 + 2

1000 + 100 + 90 + 2

As introduced in Stage 3, but moving towards more complex numbers and values. Use **place value counters** to reinforce 'exchanging'.

Compact column subtraction

6

2 7 5 4

− 1 5 6 2

1 1 9 2

To introduce the compact method, ask children to perform a subtraction calculation with the familiar partitioned column subtraction then display the compact version for the calculation they have done. Ask pupils to consider how it relates to the method they know, what is similar and what is different,

Give plenty of opportunities to apply this to money and measures.

Always encourage children to consider the best method for the numbers involved - mental, counting on, counting back or written method .

Mental strategies

A variety of mental strategies must be taught and practiced, including counting on to find the difference where numbers are closer together, or where it is easier to count on.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, **inverse**

Key skills for subtraction at Stage 4:

- Subtract by counting on where numbers are close together or they are near to multiples of 10, 100 etc.
- Children select the most appropriate and efficient methods for given subtraction calculations.
- Estimate and use inverse operations to check answers.
- Solve addition and subtraction 2-step problems, choosing which operations and methods to use and why.
- Solve simple measure and money problems involving fractions and decimals to two decimal places.
- Find 1000 more or less than a given number.
- Count backwards through zero, including negative numbers.
- Recognise place value of each digit in a 4-digit number.
- Round any number to the nearest 10, 100 or 1000.
- Solve number and practical problems that involve the above, with increasingly large positive numbers.
- Subtract numbers with up to 4 digits using a formal written method.

Subtraction

Stage 5 Subtract with at least 4-digit numbers

including money, measures, decimals.

Compact column subtraction

with 'exchanging'

Children who are still not secure with number facts and place value will need to remain on the partitioned column method until ready for the compact method.

	$^{2}\cancel{3}$	$^{10}\cancel{1}$	$^{1}0$	$^{8}\cancel{5}$	$^{1}6$
−		2	1	2	8
	2	8,	9	2	8

Subtracting with larger integers

	$^{6}\cancel{7}$	$^{10}\cancel{1}$	$^{1}6$	$^{8}\cancel{9}$	.	$^{1}0$
−		3	7	2	.	5
	6	7	9	6	.	5

Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.

Create lots of opportunities for subtracting and finding differences with money and measures.

Add a zero in any empty decimal places to aid understanding of what to subtract in that column.

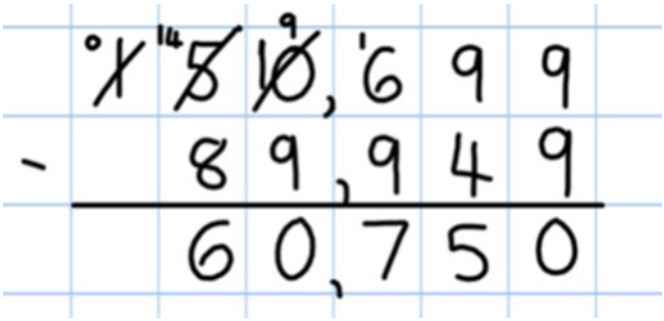
Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, inverse, **tenths, hundredths, decimal point, decimal**

Key skills for subtraction at Stage 5:

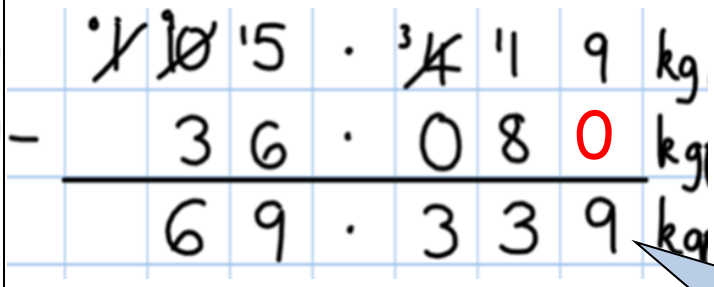
- Subtract numbers mentally with increasingly large numbers.
- Use rounding and estimation to check answers to calculations and determine, in a range of contexts, levels of accuracy.
- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Count forwards or backwards in steps of powers of 10 for any given number up to 1 million.
- Interpret negative numbers in context, counting forwards and backwards with positive and negative integers through 0.
- Round any number up to 1 million to the nearest 10, 100, 1000, 10 000 and 100 000.
- Subtract more than 4 digits using a formal written method.

Subtraction

Stage 6 Subtracting with increasingly large and more complex numbers and decimal values.



Using the compact column method to subtract more complex integers



Using the compact column method to subtract money and measures, including decimals with different numbers of decimal places.

Empty decimal places can be filled with zero to show the place value in each column.

Pupils should be able to apply their knowledge of a range of mental strategies, mental recall skills, and informal and formal written methods when selecting the most appropriate method to work out subtraction problems.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, dis- tance between, how many more, how many fewer / less than, most, least, count back , how many left?, how much less is…? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal

Key skills for subtraction at Stage 6:

- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- Round any whole number to a required degree of accuracy
- Use negative numbers in context, and calculate intervals across zero.
- Children need to utilise and consider a range of mental subtraction strategies, jottings and writ- ten methods before choosing how to calculate.

Multiplication

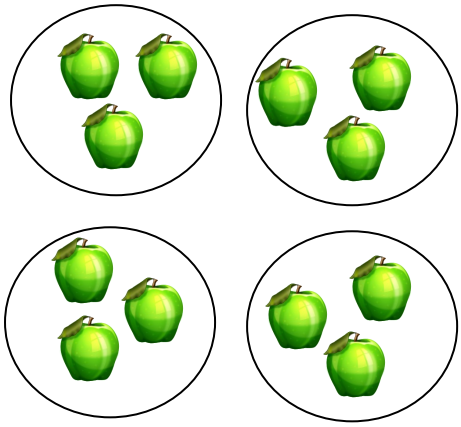
Stage 1 Multiply with concrete objects, arrays and pictorial representations

How many legs do 3 teddies have?

2 + 2 + 2 = 6

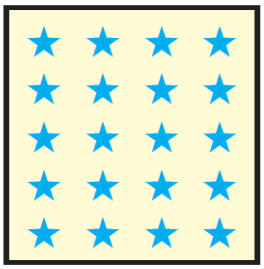
Give children experience of counting equal group of objects in 2s, 5s and 10s.

There are 3 apples in one bag. How many apples in 4 bags altogether?



3 + 3 + 3 + 3 = 12

Present practical problem solving activities involving counting equal sets or groups, as shown.



4 lots of 5 is 20

5 lots of 4 is 20

Arrays are used to support children's understanding.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count

Key skills for multiplication at Stage 1:

- Count in multiples of 2, 5 and 10.
- Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

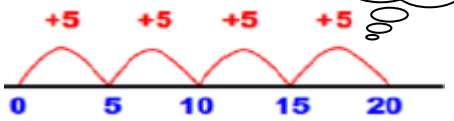
Multiplication

Stage 2 Multiply using arrays and repeated addition (using at least 2s, 5s and 10s)

Use repeated addition on a number line: 4 x 5 =...

4 lots of 5

Starting from zero, make equal jumps up on a number line to work out multiplication facts and write multiplication statements using x and = signs.



4 x 5 =20

Use arrays:

3 x 5 = 15

5 x 3 = 15

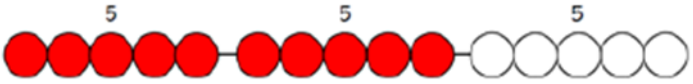
5 x 3 = 3 + 3 + 3 + 3 + 3 = **15**

3 x 5 = 5 + 5 + 5 = **15**

Use arrays to teach children to understand the commutative law of multiplication, and give examples such as 3 x ____ = 6

5 x 3 = 5 + 5 + 5

Use practical apparatus:



Use mental recall:

Children should be able to recall multiplication facts for 2, 5 and 10 times tables through practice in counting and understanding of the operation.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, **multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times...**

Key skills for multiplication at Stage 2:

- Count in steps of 2, 3 and 5 from zero, and in 10s from any number.
- Recall and use multiplication facts from the **2, 5 and 10** multiplication tables, including recognising odds and evens.
- Write and calculate number statements **using the x and = signs**.
- Show that multiplication can be done in any order (commutative).
- Solve a range of problems involving multiplication, using concrete objects, arrays, repeated addition, mental methods, and multiplication facts.
- Pupils use a variety of language to discuss and describe multiplication.

Multiplication

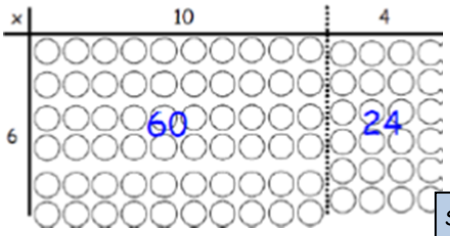
Stage 3 Multiply 2 digits by a single digit numbers

Introduce the grid method for multiplying 2 digit by single digits:

e.g. 35 x 4 = 140

x	30	5
4	120	20

Link the layout of the grid to an array initially:



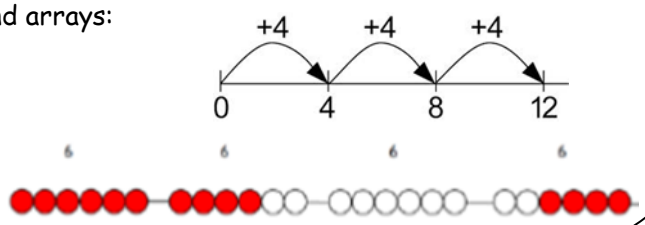
Introduce the grid method with children physically making an array to represent the calculation (e.g. make 8 lots of 23 with 10s and 1s place value counters), then translate this to grid method format.

Some children will move onto linking the grid method to the expanded written method of short multiplication (see Stage 4)

To do this, children must be able to:

- Partition numbers into tens and units
- Multiply multiples of ten by a single digit (e.g. 20 x 4) using their knowledge of multiplication facts and place value
- Recall and work out multiplication facts in the 2, 3, 4, 5, 8 and 10 times tables.
- Work out multiplication facts not known by repeated addition or other taught mental strategies (e.g. by commutative law, working out near multiples and adjusting, using doubling etc.) Strategies to support this are repeated addition using a number line, bead strings and arrays:

6 x 3 = 18



Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times, _times as big as, once, twice, three times..., **partition, grid method, multiple, product, tens, units, value**

Key skills for multiplication:

- Recall and use multiplication facts for the **2, 3, 4, 5, 8 and 10** multiplication tables, and multiply multiples of 10.
- Develop reliable written methods for multiplication, including **2-digit x single-digit**, progressing to the formal written method of short multiplication.
- Solve multiplication problems, including missing number problems.
- Develop mental strategies using commutative law (e.g. 4 x 12 x 5 = 4 x 5 x 12 = 20 x 12 = 240)
- Solve simple problems in contexts, deciding which operations and methods to use.
- Develop efficient mental methods to solve a range of problems and for missing number problems ? x 5 = 20, 3 x ? = 18

Multiplication

Stage 4 Multiply 2 and 3 digits by a single digit,

using all multiplication tables up to 12 x 12

Grid method

356 × 7

×	300	50	6	
7	2100	350	42	= 2492

Expanded written method

356 × 7

```
    3 5 6
×       7
-------
      4 2  (   6 × 7)
    3 5 0  (  50 × 7)
  2 1 0 0  (300 × 7)
-------
  2 4 9 2
```

Children are taught to look for the links between these methods so that they understand each step. They may move onto the expanded method of short multiplication when they are confident with the grid method. Some children may also move onto the compact method (see Stage 5).

Children should be able to:

- Approximate before they calculate, and make this a regular part of their calculating, going back to the approximation to check the reasonableness of their answer. e.g: 346 x 9 is approximately 350 x 10 = 3500

  Record an approximation to check the final answer against.
- Multiply multiples of ten and one hundred by a single-digit, using their multiplication table knowledge.
- Recall all times tables up to 12 x 12

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, groups of, sets of, lots of, equal groups, times, multiply, times, as big as, once, twice, three times... partition, grid method, total, multiple, product, sets of, **inverse**

Key skills for multiplication at Stage 4:

- Count in multiples of 6, 7, 9, 25 and 1000
- Recall multiplication facts for **all multiplication tables up to 12 x 12**.
- Recognise place value of digits in up to 4-digit numbers
- Use place value, known facts and derived facts to multiply mentally, e.g. multiply by 1, 10, 100, by 0, or to multiply 3 numbers.
- Use commutative law and other strategies mentally 3 x 6 = 6 x 3 , 2 x 6 x 5 = 10 x 6 , 39x7 = 30 x 7 + 9 x 7.
- Solve problems with increasingly complex multiplication in a range of contexts.
- Solve problems involving scaling e.g. if 2 children have 3 cakes, how many will 6 children have?
- Multiply 2 and 3 digit numbers by a 1-digit number using a written method.

Multiplication

Stage 5 Multiply up to 4 digits by 1 or 2 digits

Children need to be taught to approximate first e.g. for 93 x 58 they will use rounding. 93 x 58 is approximately 90 x 60 = 5400; use the approximation to check the reasonableness of their answer.

Short multiplication for multiplying by a single digit

Expanded written method

378 × 4

```
    3 7 8
×       4
-----------
      3 2   (   8 × 4)
    2 8 0   (  70 × 4)
  1 2 0 0   (300 × 4)
-----------
  1 5 1 2
-----------
      1
```

Formal written method of short multiplication

378 × 4

```
    3 7 8
×   3 3 4
-----------
  1 5 1 2
```

Children will be encouraged to taught to use the expanded written method for short multiplication. Some will move on to the compact method. What are the similarities and differences? Unpick the steps and show how it reduces the number of stages.

Introduce long multiplication for multiplying by 2 digits

×	40	8
60	2400	480
4	160	32

```
   64
X  48
   32
  480
  160
 2400
 3072
```

Teach the children to multiply by ten first when working in the tens column (put a zero in the units) then think 4 x 4 = 16, 4 x 60 = 240.

If they are confident with the compact method of short multiplication, they may also move onto the compact method of long multiplication (Stage 6).

Use grid method to introduce long multiplication, as the individual steps can be clearly seen.

Children should move on to more complex calculations (e.g. 1436 x 18) as they become more confident.

Measures and money should be used regularly as context for calculations.

Key vocabulary groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, _times as big as, once, twice, three times..., partition, grid method, total, multiple, product, inverse, **square, factor, integer, decimal, short/long multiplication, 'carry'**

Key skills for multiplication at Y5:

- Identify multiples and factors, using knowledge of **multiplication tables to 12 x 12.**
- Solve problems where larger numbers are decomposed into their factors.
- Multiply and divide integers and decimals by 10, 100 and 1000.
- Recognise and use square and cube numbers and their notation.
- Solve problems involving combinations of operations, choosing and using calculations and methods appropriately.
- Multiply numbers up to 4 digits by a 1 or 2-digit number using a formal written method.

Multiplication

Stage 6 Short and long multiplication, and multiply decimals with up to 2 d.p. by a single digit

Short multiplication: move from extended to compact method

```
 148
X  4
  32
 160
 400
 592
```

```
 148
X  4
 592
 1 3
```

In the units column, 4 x 8 = 32. Write the 2 in the units column and 'carry' the 3 into the tens.
In the tens column, 4 x 4 = 16 plus 3 tens carried in makes 19. Write 9 in the tens column and 'carry' 1 into the hundreds column.
In the hundreds column, 4 x 1 = 4 plus the 1 hundred carried from the tens column makes 5.

Long multiplication: move from extended to compact method

```
  64
X 48
  32
 480
 160
2400
3072
```

```
  64
X 48
 512
2560
3072
```

64 x 8 on the first row
In the units column, 8 x 4 = 32: write down the 2 and carry the 3
In the tens column, 8 x 6 = 48 plus the 3 carried in makes 51.
64 x 40 on the second row
Multiply the answer by 10 first by putting a zero in the units column which moves all the digits one place to the left.
In the tens column, 4 x 4 = 16; write down the 6 and carry the 1
In the hundreds column, 4 x 6 = 24 plus the 1 carried in makes 25

Multiplying with decimal numbers

```
   3 . 1 9
x  8
2 5 . 5 2
  1   7
```

Remind children:

- Line up the decimal points in the question and the answer.
- The single digit belongs in the units column.

Children should be able to:

- Use rounding and place value to approximate and check answers.
- Use short multiplication to multiply numbers with more than 4 digits, including numbers with up to 2 d.p. by a single digit.
- Use long multiplication to multiply numbers with at least 4 digits by 2 digits.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times… partition, grid method, total, multiple, product, inverse, square, factor, integer, decimal, short / long multiplication, 'carry', **tenths, hundredths, decimal**

Key skills for multiplication at Stage 6:

- Recall multiplication facts for all times tables up to **12 x 12 (as Y4 and Y5).**
- Multiply multi-digit numbers, up to 4-digit x 2-digit using long multiplication.
- Multiply multi-digit numbers by 1-digit using short multiplication.
- Perform mental calculations with mixed operations and large numbers.
- Solve multi-step problems in a range of contexts, choosing appropriate combinations of operations and methods.
- Estimate answers using round and approximation and determine levels of accuracy.
- Round any integer to a required degree of accuracy.

Division

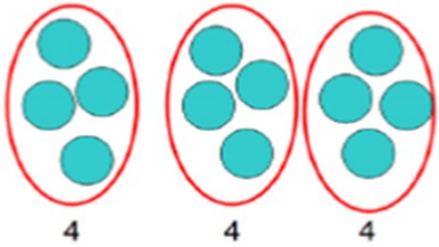
Stage 1 Group and share small quantities

Using objects, diagrams and pictorial representations to solve problems involving **both grouping and sharing**.

Grouping

How many groups of 4 can be made with 12 stars? = **3**

Sharing



12 shared between 3 is 4

Example division problem in a familiar context:

There are 6 pupils on this table and there are 18 pieces of fruit to share between us. If we share them equally, how many will we each get?

Can they work it out and give a division statement… ?

"18 shared between 6 people gives you 3 each."

Pupils should:

- Use lots of practical apparatus, arrays and picture representations.
- Be taught to understand the difference between 'grouping' objects (How many groups of 2 can you make?) and 'sharing' (Share these sweets between 2 people).
- Be able to count in multiples of 2s, 5s and 10s.
- Find half of a group of objects by sharing into 2 equal groups.

Key Vocabulary: **share, share equally, one each, two each…, group, groups of, lots of, array**

Key number skills needed for division at Stage 1:

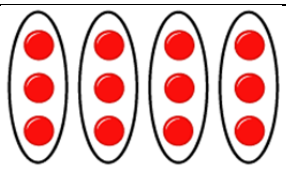
- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations arrays with the support of the teacher
- Through grouping and sharing small quantities, begin to understand, division, and finding simple fractions of objects, numbers and quantities.
- Make connections between arrays, number patterns, and counting in twos, fives and tens.

Division

Stage 2 Group and share using the ÷ and = sign

Use objects, arrays, diagrams, physical resources and pictorial representations to explore sharing and grouping. Use a number line.

Arrays:



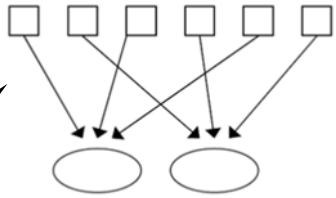
12 ÷ 3 = 4

This represents 12 ÷ 3, posed as how many groups of 3 are in 12?

Pupils should also show that the same array can represent 12 ÷ 4 = 3 if grouped horizontally.

Know and understand sharing and grouping

6 sweets shared between 2 people; how many do they each get?



Sharing

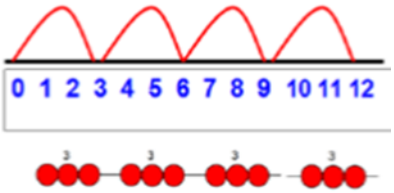
Grouping

There are 6 sweets, how many people can have 2 sweets each?

Children are taught to recognise whether problems require sharing or grouping.

Grouping using a number line

Group from zero in equal jumps of the divisor to find out 'how many groups of _ in _ ?'. Pupils could use a bead string or practical apparatus to work out problems like 'A CD costs £3. How many CDs can I buy with £12?' **This is an important method to develop understanding of division as grouping.**



12 ÷ 3 = 4

Pose 12 ÷ 3 as 'How many groups of 3 are in 12?'

Key Vocabulary: share, share equally, one each, two each…, group, groups of, lots of, array, **divide, divided by, divided into, division, grouping, number line, left, left over**

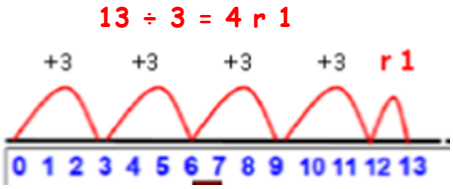
Key number skills needed for division at Stage 2:

- Count in steps of 2, 3 and 5 from 0
- Recall and use multiplication and division facts for the **2, 5 and 10** multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the x, ÷ and = signs.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

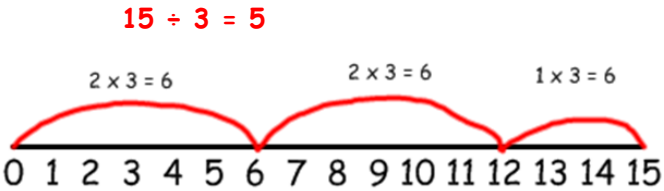
Division

Stage 3 Divide 2 digit numbers by a single digit

(where there is no remainder in the final answer)

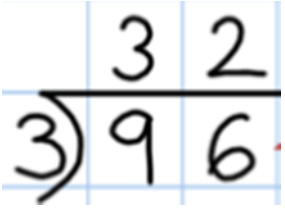


STEP 1: Children continue to work out unknown division fact by grouping on a number line from zero. They are also now taught the concept of remainders, as in the example. This should be introduced practically and with arrays, as well as being translated to a number line.



STEP 2: Children begin to use 'chunks' of the divisor to speed up the process.

Short division: Limit numbers to NO remainders in the answer OR carried (each number must be a multiple of the divisor)



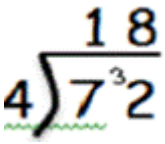
STEP 3: Once the children are secure with division as grouping and can demonstrate this using number lines, short division for larger 2 digit numbers should be introduced, initially with carefully selected examples requiring no calculation of remainders at all.

Remind children of correct place value (96 is 90 + 6) but work across each column:

In the 10s column, how many 3s in 9? Record in 10s column.

In the 1s column, how many 3s in 6? Record in 1s column.

Short division: Limit numbers to NO remainders in the final answer, but remainders occurring within the calculation.



STEP 4: Once children demonstrate an understanding of the short division method, introduce the concept of remainders with examples where the remainder occurs within the calculation (e.g. 72 ÷ 4). Teach the children to 'carry' the remainder to the next digit.

Real life contexts should be used routinely to help pupils to recognise division and understand how to apply it to problems.

Only introduce Step 4 when children can calculate remainders mentally.

Key Vocabulary: share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, **inverse, short division, 'carry', remainder, multiple**

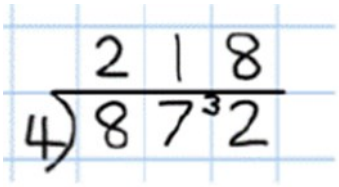
Key number skills needed for division at Stage 3:

- Recall and use multiplication and division facts for the **2, 3, 4, 5, 8 and 10** multiplication tables (through doubling, connect the 2, 4 and 8s).
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
- Solve problems, in contexts, and including missing number problems, involving multiplication and division
- Pupils develop efficient mental methods, for example, using multiplication and division facts (e.g. using 3 × 2 =6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3) to derive related facts (30 × 2 = 60, so 60 ÷ 3 = 20 and 20 = 60 ÷ 3).
- Pupils develop reliable written methods for division, starting with calculations of 2-digit numbers by 1-digit numbers and progressing to the formal written method of short division.

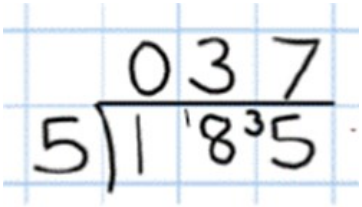
Division

Stage 4 Divide up to 3 digit numbers by a single digit (without remainders initially)

Continue to develop short division:



Once children are secure with Stage 3 expectations, they can move onto dividing numbers with up to 3 digits by a single digit. Problems should not result in a final remainder at this stage. Children who exceed this expectation may progress to Stage 5.



When the answer for the first column is zero (e.g. 1 ÷ 5), children could write a zero above to acknowledge the place value. The number must be carried over to the next digit as a remainder.

Include money and measures contexts when children are confident with the short division method.

Key Vocabulary: share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, **divisible by, factor**

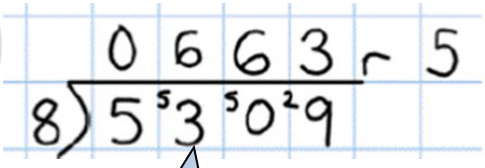
Key number skills needed for division at Stage 4:

- Recall multiplication and division facts for all numbers up to **12 x 12**.
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1.
- Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number.
- Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example 200 × 3 = 600 so 600 ÷ 3 = 200
- Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

Division

Stage 5 Divide up to 4 digit numbers by a single digit, including those with remainders

Short division, including remainder answers:



Short division with remainders: Now that children are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where **pupils consider the meaning of the remainder and <u>how</u> to express it,** i.e.. as a fraction, a decimal, or as a rounded number or value , depending upon the context of the problem.

The answer to 5309 ÷ 8 could be expressed as 663 $^{5}/_{8}$, 663 r 5, 663.625, 663 or 664 depending on the problem involved.

Continuing short division to give a decimal answer is covered in Stage 6.

If children are confident and accurate with short division, introduce **long division** - dividing by a 2 digit number. This is a Stage 6 expectation.

Key Vocabulary: share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor, inverse, **quotient, prime number, prime factors, composite number (non-prime)**

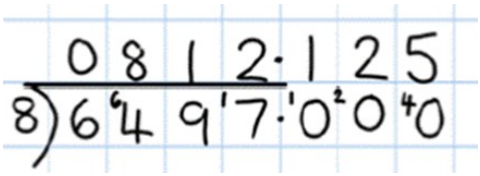
Key number skills needed for division at Stage 5:

- Multiply and divide numbers mentally, drawing upon known facts.
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two number.
- Solve problems involving multiplication and division where larger numbers are decomposed into their factors.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Work out whether a number up to 100 is prime, and recall prime numbers to 19.
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.
- Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (e.g. 98 ÷ 4 = 24 r 2 = 241/2 = 24.5 ≈ 25).
- Use multiplication and division as inverses.
- Solve problems involving combinations of all four operations, including understanding of the equals sign, and including division for scaling by different fractions and problems involving simple rates.

Division

Stage 6 Divide at least 4 digit numbers by both single digit and 2 digit numbers (including decimal numbers and quantities)

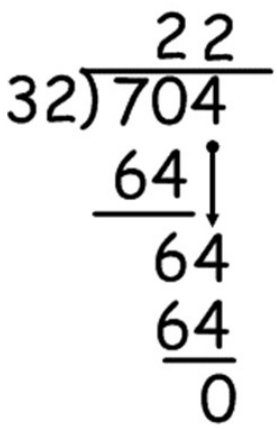
Short division, for dividing by a single digit e.g. 6497 ÷ 8



Short division with remainders: Children should continue to use this method, but with numbers to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers. Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.

Calculating a decimal remainder: In this example, rather than expressing the remainder as r 1, a decimal point is added after the units because there is still a remainder, and the one remainder is carried on to zeros after the decimal point (to show there was no decimal value in the original number). Keep dividing to an appropriate degree of accuracy for the problem being solved.

Introduce long division for dividing by 2 digits using expanded method



Teach the children to work across the place value columns, asking themselves each time 'What is the biggest multiple of the divisor I can take away?'. Write the size of the 'chunk' of the divisor on the answer line, then subtract the multiple. Once the subtraction is completed, bring down the next digit from the answer and repeat the process. As with short division, answers can be given as a whole number with a remainder, as a mixed number or with a decimal depending on the context.

Using mental strategies e.g. repeated doubling will speed up finding the 'times table' of the divisor.

Key Vocabulary: share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor, inverse, quotient, prime number, prime factors, composite number (non-prime), **common factor**

Key number skills needed for division at Stage 6:

- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Use short division where appropriate.
- Perform mental calculations, including with mixed operations and large numbers.
- Identify common factors, common multiples and prime numbers.
- Solve problems involving all 4 operations.
- Use estimation to check answers to calculations and determine accuracy, in the context of a problem.
- Use written division methods in cases where the answer has up to two decimal places.
- Solve problems which require answers to be rounded to specified degrees of accuracy.